New Structural Form of Sandwich Core

Koryo Miura*
University of Tokyo, Tokyo, Japan

The purpose of this study is to present a new structural form of sandwich core. The observation, that the characteristics of honeycomb core is distinctly attributable to its stabilized perpendicular wall elements, leads to the possibility of a hypothetical core concept characterized by stabilized oblique wall elements. To embody the hypothetical concept, the core form constituted by superposing two mutually orthogonal corrugations is proposed. The resultant core can be manufactured from a single sheet by some press forming technique. The theoretical and experimental analyses of the core reveal that the shear modulus and strength are comparable with those of honeycomb core, and the elastic properties can be designed to be either isotropic or orthotropic. Other features of the core such as the simplicity of form, the applicability to both flat and curved sandwiches, and the possibility of circulating fluid between facings, may excite new applications of sandwich construction in aircraft structures.

Nomenclature

a = area of rhomboidal element
E = Young's modulus
g,h = periodic functions
G = shear modulus
l,m,n = components of unit normal vector
r = unit vector
t = uniform thickness of rhomboidal element
U = strain energy
v = gross volume between facings filled by a fundamental region
x,y,z = rectangular Cartesian coordinates
α = filling factor of core
β = reduction ratio [see Eq. (8)]
γ = shear strain
ϵ = normal strain
θ = inclination angle of oblique element
μ = form efficiency of core [see Eq. (7)]
σ = normal stress
ν = Poisson's ratio
τ = shear stress
ψ = direction, ψ radian counter-clockwise from x-axis on x-y plane
ω = zigzag angle of ridge

Superscript
$*$ = quantity relating to a zeta-core whose fundamental region is composed of four congruent rhomoids and two congruent chevron pattern (see Fig. 4)

Subscripts
av = average
c = core property
i = ith element
iso = isotropic
y = yield

I. Introduction

WITH the advancement of sophisticated design of aircraft structures, the use of structural sandwich construction is extending steadily. At present the honeycomb sandwich seems to be the only accepted configuration in aircraft application due to its high structural effificiency and reliability. However, as the requirements of sandwich diversifies to meet with various functions and environments, the honeycomb sandwich should not always be considered as the only answer. Instead, other core configurations should either be sought among known forms or created in the three-dimensional space. The purpose of this study is to create such a new structural form of the sandwich core usable at higher stress levels. This paper is written along the three sequential steps of the research on the subject: the proposition of a hypothetical core concept, the embodiment of the concept, and the analytical and experimental studies on the elastic properties of the proposed core.

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*Professor, Institute of Space and Aeronautical Science, University of Tokyo, Meguro, Tokyo. Member AIAA.

II. Emerging of a New Core Form

From the point of view of structural concept, the excellent characteristic of honeycomb core is distinctly attributable to its stabilized perpendicular wall elements in proper orientation and distribution. This observation raises a question whether the wall elements should necessarily be perpendicular to facings. Since there is no reason why the question is to be answered in the affirmative, the possibility of a core having oblique wall elements cannot be excluded.

To obtain a key to the solution of the question, we shall consider a hypothetical core made of nonperpendicular, oblique wall elements as shown in Fig. 1a. This core is formed by orthogonally intersecting, in mathematical sense, a pair of v-corrugated sheets so that the crest ridges of the sheets form a square lattice. This is compared with the honeycomb core of square type shown in Fig. 1b, which is assumed to have the identical apparent density with the hypothetical core.

For purposes of comparison, let us calculate the strain energy of each core per unit gross volume when it constitutes a sandwich construction and is subject to a certain amount of plate shear deformation. Within the scope of the approximate membrane stress analysis, the ratio of strain energies of oblique core to perpendicular core is $\sin^2\theta + (\sin^2 2\theta)/2$, where θ is the angle between oblique walls and facings. Apparently this value slightly exceeds 1 for $\pi/4 < \theta < \pi/2$. Thus it can be concluded that a hypothetical core having oblique wall elements can be more rigid than the honeycomb core. Although manufacturing the hypothetical core is rather difficult, the previous conclusion indicates the potentialities of this kind of core if it can be embodied in some other forms.

During the study of deformation of an infinite plate, the present author has been aware of the existence of the doubly corrugated surface.[1,2] It is the surface made by superposing ingeniously two corrugations in mutually orthogonal directions. As shown in the relief of Fig. 2, a typical double

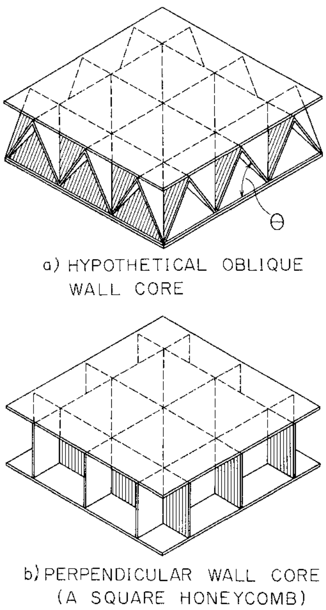

Fig. 1 Sandwich constructions with oblique and perpendicular wall core.

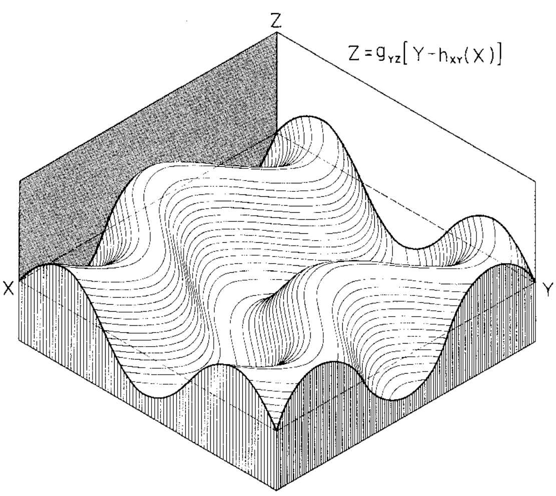

Fig. 2 Double corrugation surface.

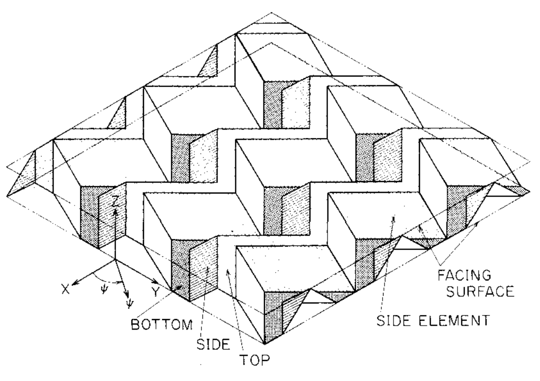

Fig. 3 Zeta-core, a embodiment of hypothetical oblique wall core concept.

corrugation surface can be expressed by the function

$$z = g_{yz}[y - h_{xy}(x)] \tag{1}$$

where g_{yz} and h_{xy} represent certain continuous, single-valued, bounded, periodic functions defined in y-z and x-y plane, respectively.

At this point, a little insight into the problem will bring us from the configuration of Fig. 2 to a configuration shown in

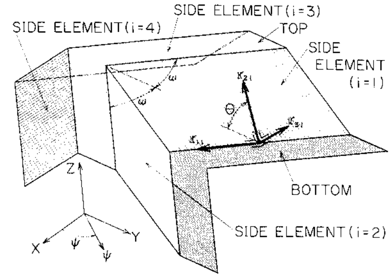

Fig. 4 Fundamental region of a zeta-core.

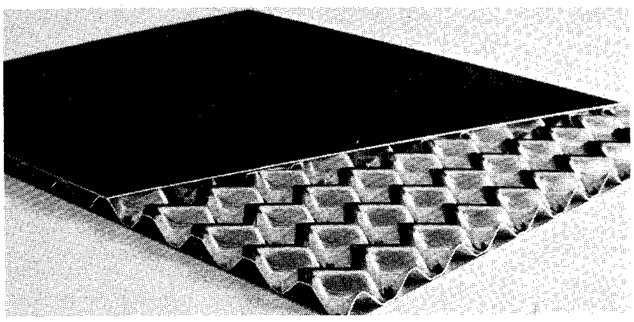

Fig. 5 Aluminum zeta-core sandwich.

Fig. 3, which is a real embodiment of the aforementioned hypothetical oblique wall core concept.[3] Geometrically this surface is constructed by using a truncated zigzag function $g_{yz}(y)$ and a zigzag function $h_{xy}(x)$. It can also be constructed by the repetition of the fundamental region which is composed of four congruent rhomboids and two congruent chevron patterns as shown in Fig. 4. In this configuration, it is seen that each rhomboidal element of the sides of corrugations is supported by adjacent elements and facings. Therefore, it provides a stabilized structural component workable under various loading conditions. The diverse orientation of each rhomboidal element can control the macroscopic properties of the sandwich construction either isotropic or orthotropic. In addition, zigzag and interlocking top and bottom parts of the core can provide proper and well distributed bonding zones. The combinations of other functions g and h give many variations in core forms.

It may be convenient to have an adequate designation for the core thus defined. Since both capital and small letters of Greek alphabet Z and ζ remind us of periodic functions g and h or an overall impression about the core, the English pronounciation of this letter, zeta, is adopted here like as zeta-core. A sample of an aluminum zeta-core sandwich is shown in Fig. 5.

III. Elastic Properties of Zeta-Core

Among the principal quantities relating to the elastic properties of core, the shear modulus G_c, or more specifically the effective modulus of rigidity of core in planes including normals to facings, is an important as well as only representative quantity. The shear modulus shall now be calculated analytically for a zeta-core whose configuration is shown in Fig. 3.

The rectangular coordinates x,y,z are taken so that x and y axes may lie on the facing surface. Further, the direction ψ is defined, which is ψ radian counter-clockwise from x axis on x-y plane. The strains of the core are now considered when the sandwich is subject to a shear deformation $\gamma_{\psi z}$ in ψ-z plane. The stress and strain distribution in the oblique side wall elements of rhomboidal form is undoubtedly very complicated. Because the present aim of the computation is to obtain a macroscopic elastic quantity with regards to shear property, and not to get detailed local stress distribution within core elements, some adequate approximation can be made without impairing the essence of the problem.

First, it is assumed that the facings are infinitely rigid as for both in-plane and bending deformation. Secondly, the presence of the membrane stress state is assumed. Then, for an arbitrary element i (Fig. 4), the deformation applied externally from facings is the relative parallel transfer of a pair of edges on facings. Therefore, the first approximation on the strain distribution, which can be compatible with the displacements at facings, is the homogeneity of strain within the element i. If this situation is accepted, the resulting strains in each element are identical to the strains of the identical position and direction in a hypothetical core made of a homogeneous continuum subjected to the said external displacements.

Now, a normal unit vector $r_{3i}\{l_i, m_i, n_i\}$ is defined on the ith rhomboidal element as shown in Fig. 4. Also, a couple of vectors r_{1i} and r_{2i} are defined, r_{1i} on the intersection of the x-y plane and the ith element, r_{2i} vertical to r_{1i} on ith element. The normal strains of the element i are represented by the extensional strains ϵ_{1i} and ϵ_{2i} of the vectors r_{1i} and r_{2i}, respectively.

$$\epsilon_{1i}=0 \tag{2}$$

$$\epsilon_{2i}=\gamma_{\psi z}(-m_i n_i \sin\psi - l_i n_i \cos\psi) \tag{3}$$

The shear strain γ_{12i} due to $\gamma_{\psi z}$ can be calculated as the shear deformation of two vectors r_{1i} and r_{2i} which were originally perpendicular to each other before deformation.

$$\gamma_{12i}=\gamma_{\psi z}(-l_i \sin\psi + m_i \cos\psi) \tag{4}$$

The total strain energy of the core is a simple summation of the partial strain energy for each element and is given by

$$U=\tfrac{1}{2}\gamma_{\psi z}^2 \sum_i a_i t_i [G(-l_i\sin\psi+m_i\cos\psi)^2 + E(1-\nu^2)^{-1}(m_i n_i \sin\psi + l_i n_i \cos\psi)^2] \tag{5}$$

where a_i and t_i represent the area and the constant thickness of the element. And the summation is carried out within the fundamental region. Now, the gross volume between facings filled by a fundamental region of a zeta-core is denoted by v. The strain energy U of the hypothetical continuum of the same gross volume v and with the shear modulus G_c, when it is subject to a uniform shear strain $\gamma_{\psi z}$, is $\gamma_{\psi z}^2 v G_c/2$. Because this must be equal to the Eq. (5), the effective shear modulus of a zeta-core is given by the following formula.

$$G_c=2U/\gamma_{\psi z}^2 v \tag{6}$$

Hereupon the concept of form efficiency of core is introduced that is a dimensionless form of shear modulus defined by

$$\mu=G_c/G\alpha \tag{7}$$

where G is the shear modulus of the material of core, and α is the spatial filling factor of core. In addition, the following reduction ratio is defined to exclude the flat top (bottom) part of the core from the computation, because this dead volume does not contribute to the strain energy.

$$\beta=[(\text{net volume})-(\text{volume of top \& bottom})]/(\text{net volume}) \tag{8}$$

From Eqs. (5, 6, 7, and 8) the form efficiency of the zeta-core is given by the following formula.

$$\mu(\psi)=(\beta/\sum_i a_i t_i)\sum_i a_i t_i[G(-l_i\sin\psi+m_i\cos\psi)^2 + E(1-\nu^2)^{-1}(m_i n_i\sin\psi+l_i n_i\cos\psi)^2] \tag{9}$$

In case of the fundamental region of Fig. 3, if an appropriate rectangular coordinates x,y,z are chosen like those in the figure, the components of normal unit vectors of four rhomboidal elements composing a fundamental region can be expressed solely by a set of l^*, m^*, n^*, which are assigned to be positive. By this simplified manipulation, Eq. (9) reduces to

$$\mu(\psi)=\beta[(l^{*2}+\frac{2m^{*2}n^{*2}}{1-\nu})\sin^2\psi + (m^{*2}+\frac{2l^{*2}n^{*2}}{1-\nu})\cos^2\psi] \tag{10}$$

Since the design of sandwich constructions is primarily effected by the elastic property of the weakest direction, the isotropy of the core property is generally desirable. In case of the previous example, the circular isotropy condition is realizable when the following relation is satisfied between components of normal unit vectors of rhomboidal elements,

$$l^{*2}+2m^{*2}n^{*2}/(1-\nu)=m^{*2}+2l^{*2}n^{*2}/(1-\nu) \tag{11}$$

thus, by virtue of Eq. (10), the μ value of the core is independent of ψ value. There are two cases that satisfy above relation and these are:

Case I $\quad l^*=m^*$

$$\mu_{\text{iso}}=\beta\{\tfrac{1}{2}+(1+\nu)n^{*2}/[2(1-\nu)]-n^{*4}/(1-\nu)\} \tag{12}$$

Case II $\quad n^*=[(1-\nu)/2]^{1/2}$

$$\mu_{\text{iso}}=\beta(1+\nu)/2 \tag{13}$$

In practice, however, it is not necessary to provide a complete isotropy. Rather, it is preferred to leave some room for selection so that other design requirements such as the ease of manufacturing or requirements for ultimate use can be met. In view of these, it is preferred to allow a deviation from complete isotropy within the range of ±20%. In Fig. 6, the inclination angle of element, $\theta=\cos^{-1}n^*$, is plotted on the axis of abscissa, and $m^*/1$ on the axis of ordinate. The combination of 1^*, m^*, and n^* which gives complete isotropy is shown by line I where $m^*=1^*$ and line II where $n^*=[1-\nu)/2]^{1/2}$. Contour line-like curves drawn with these lines as axes show the case of deviation of ±5%, ±10%, ±15%, and ±20% sequentially from the center, and the region which

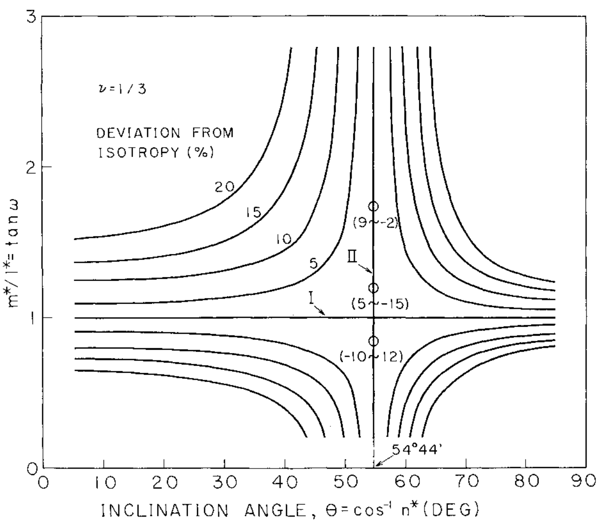

Fig. 6 Combination of orientation variables which gives isotropic elastic property.

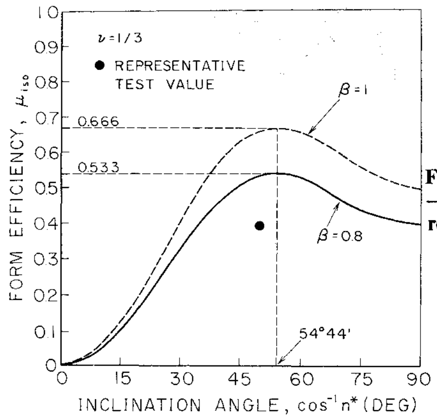

Fig. 7 Form efficiency —inclination angle relation.

shows a substantial isotropy is shown by the criss-cross region. But not all of the region is suitable for actual designing because of either production difficulty or inherent inferior property. Therefore, the design envelope may be further restricted.

As a matter of course, the core desirably has a large μ value besides being istropic. For example, in case of honeycomb core, the μ value is 0.625 for the ribbon direction and 0.375 for the other principal direction. Provisionally these values can be the target for high strength cores.

It should be noted that in the Case I isotropy configuration, μ value is the function of the component n^* alone. Figure 7 is a graphical representation of μ-n^* relation. The maximum value of μ_{iso} is given by the following formula:

$$\mu_{iso} = \beta\{ 1/2 + (1+\nu)/[16(1-\nu)]\}, \text{ at } n^* = (1+\nu)^{1/2}/2$$
$$= \beta(2/3), \qquad \text{if } \nu = 1/3 \qquad (14)$$

If a typical value of β is assumed to be 0.8, then $\mu_{iso} = 0.533$ for the inclination angle $\theta = \cos^{-1} n^* = 54°44'$ when $\nu = 1/3$.

It is interesting to note that in Case II isotropy, the μ value is constant,

$$\mu_{iso} = \beta(1+\nu)/2 \quad \text{at } n^* = [(1-\nu)/2]^{1/2} \qquad (15)$$
$$= \beta(2/3) \qquad \text{if } \nu = 1/3$$

and it is almost coincident with the maximum value for Case I isotropy.

If Fig. 7 is seen with reference to Fig. 6, combinations of l^*, m^*, and n^* and the corresponding μ value are clearly understood. As is seen from Eq. (14), μ/β is maximum at 2/3 when $n^* = (1+\nu)^{1/2}/2$ ($= 1/3^{1/2}$ when $\nu = 1/3$). This almost coincides with the conditions for complete isotropy, that is $n^* = [(1-\nu)/2]^{1/2}$ ($= 1/3^{1/2}$ when $\nu = 1/3$). Generally, the region defined to provide isotropy corresponds with the region having a large μ. In other words, this defined region shows double peaks of the scale of isotropy and the scale of the absolute value of μ.

Some preliminary experiment was done using cores made of polystylene sheets. The cores of Fig. 3 configuration

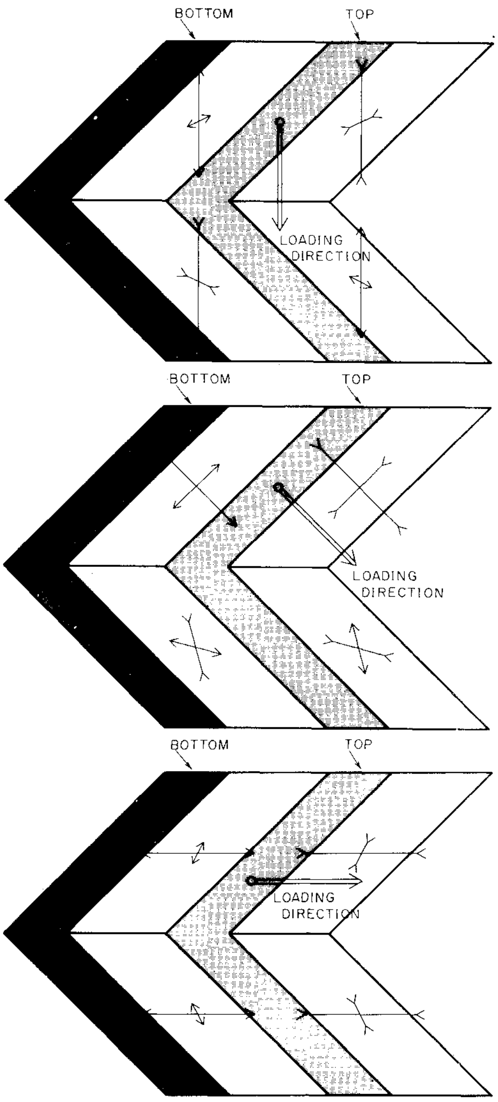

Fig. 8 Projected principal stresses in oblique wall elements of a zeta-core subject to shear force (Loading direction indicates relative displacement of top facing to bottom facing).

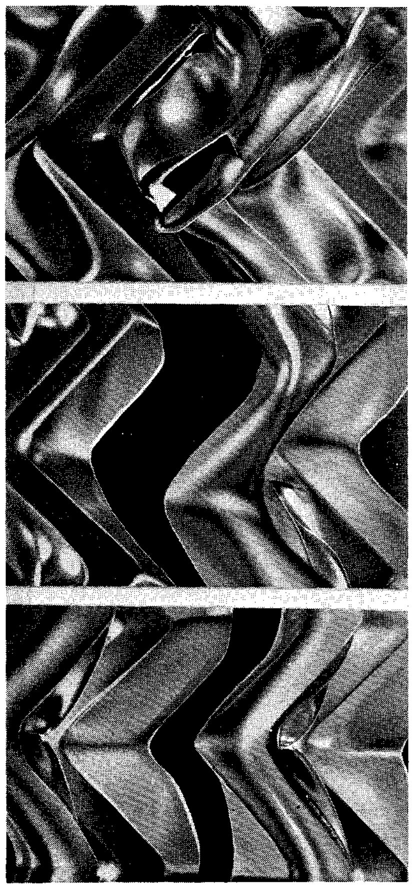

Fig. 9 Core fracture modes corresponding to loading conditions specified in Fig. 8.

corresponding to Case II isotropy ($\theta = 54°44'$) and having different values of zigzag angle ω were tested as for the form efficiency in x and y directions. The resulting μ_x/μ_{av} and μ_y/μ_{av} values are plotted in Fig. 6 at the side of circular points representing orientation variables.

Another preliminary test was done using Case I isotropy ($1^* = m^*$) cores ($\beta = 0.8$) on purpose of obtaining the absolute value of the form efficiency. The result shown in Fig. 7 as solid circular point indicates that the tested value is about 25% less than the theoretical value supposing infinitely rigid adhesive layers.

IV. Shear Strength of Zeta-Core Sandwich

It seems to us that the strength property of a sandwich construction can be represented by its plate shear strength. In connection with this, from the standpoint of the purpose of the present paper, the following data are needed; the stresses in the core wall, the fracture mechanism of the core assuming the facings are infinitely rigid, and the shear strength value in comparison with other cores. A limited number of such data are shown in the following about a typical configuration of the zeta-core sandwich.

The stress formulas in rhombic wall elements can be obtained from Eqs. (2-4) as follows:

$$\sigma_{1i} = [E\nu\gamma_{\psi z}/(1-\nu^2)](-m_i n_i \sin\psi - l_i n_i \cos\psi)$$

$$\sigma_{2i} = [E\gamma_{\psi z}/(1-\nu^2)](-m_i n_i \sin\psi - l_i n_i \cos\psi) \qquad (16)$$

$$\tau_{12i} = [E\gamma_{\psi z}/2(1+\nu)](-l_i \sin\psi + m_i \cos\psi)$$

The more detailed analysis using the FEM indicates that the above formulas represent the right reature of stress states. As for the fundamental region of the example configuration (Fig. 4), the principal stresses are different from one another, because these elements make different angles against the applied force direction. The result of the computation is shown in the plane figure of the fundamental region (Fig. 8). In this figure, the segments representing principal stresses are directly scribed on the oblique (54°44′) wall and then projected on the plane figure. Therefore, the two principal stresses apparently do not intersect at right angles in the figure. It is observed that the stress is dependent on both the orientation of the core element and the direction of loading, and that there are elements principally dominated either by tensile stress, or compressive stress, or shear stress. Such stress state offers a marked constrast with that of the honeycomb core where the simple shear stress state prevails within its vertical cell wall.

The fracture of the core should undoubtedly be effected by such stress state, and it was fully supported by the fracture test of the aluminum zeta-core sandwiches. Figure 9 shows the plane view of the aspect of core fracture which corresponds to the specified loading direction indicated in adjacent Fig. 8.

The shear strength of the same specimen is shown in Fig. 10 in a dimensionless form using the yield strength of the used material and the filling factor of the core. For comparison purposes, the typical value of the honeycomb sandwich is plotted also in the same figure.

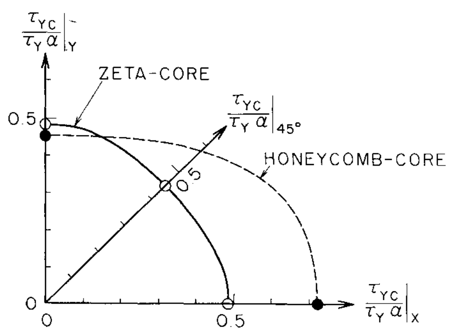

Fig. 10 Shear strength of a zeta-core in principal directions.

V. Problems in Manufacturing and Application

The manufacturing of zeta-core is generally not easy but is within the reach of conventional sheet forming techniques. It can be made by the progressive die press from a flat sheet metal. The amount of in-plate plastic deformation is an increasing function of the inclination of wall element of core, and the difficulty increases as this angle increases. The angle near 55° which gives the maximum shear modulus is a goal. As is easily imagined, this implies a large amount of in-plane deformation which almost exceeds the limit of a ductile aluminum sheet.

This limitation in manufacturing, therefore, has an effect on designing the detailed form of core. In the present case, however, the flat top part of the crest ridges of the corrugations, which is intentionally provided for the bonding area, has turned to be beneficial from the standpoint of manufacturing. In fact, the flat top part of the core has an eminent alleviating effect on the excessive straining of sheet. Without this flat part, either excessive thinning or fracture in the region of sharp crest ridges of the core is almost inevitable. It is expected that within a few years of developing endeavors, the manufacturing technique of metallic zeta-core will achieve much in both its quality and productivity.

Hereupon, a word or two may be devoted to the application of the zeta-core sandwich. Some applications may benefit by the following features of the zeta-core sandwich: a) high shear modulus and strength, b) isotropy or controllable orthotropy of shear modulus, c) having a continuous single core structure, d) applicability to cylindrical configuration, e) easiness in bonding, f) possibility of circulation of fluids within the core. On the other hand, the following demerits are enumerated: a) the greater free span of facings between core walls for the core of greater depth, b) incapability of cutting it into versatile shapes, for example, a tapered shape, c) lack of experience.

VI. Conclusions

The observation, that the characteristics of honeycomb core is distinctly attributable to its stabilized perpendicular wall elements, led to the possibility of a hypothetical core concept characterized by stabilized oblique wall elements. The concept was realized by the core form constituted by superposing two mutually orthogonal corrugations. The theoretical and experimental analyses of the core revealed that the shear modulus and strength are comparable with those of honeycomb core and the elastic properties can be controlled to either isotropic or orthotropic. Other features of the core include the simplicity of form, the applicability to both flat and curved sandwiches, the possibility of circulating a fluid between facings. It appears well established that the proposed sandwich core concept potentially adds to our stock of structural concepts in aircraft applications.

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